Measuring Efficiency with a Linear Economic Model

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Abstract – This paper modifies and interprets Data Envelopment Analysis (DEA) using a linear economic model. This approach is similar to the cone input/output and assurance region approaches to DEA, but it is implemented so that the multipliers are measured in the same units across all linear optimization problems. This approach allows us to interpret alternatives as profit maximizing organizations and the DEA multipliers as prices that are comparable across the alternatives. This is a useful extension of the assurance region concept, but more important, is that our approach enhances communication with decision-makers. The improved communication is illustrated by applying the model to the siting of a long-term health care facility. This application is interesting because the multiplier bounds make practical sense, and because the problem has dimensions that sometimes lead to interpretation problems with the traditional DEA model. For example, the site characteristics do not result from coordinated decisions, some sites exhibit zero values for some variables, and the problem has many variables compared with the number of potential sites. Problems with these dimensions have, at times, been deemed unsuitable for DEA, but they are handled without problem by the linear economic model.

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1. Introduction

The goal of constructing a simple measure of economic productivity has a long and interesting history. Measures as diverse as the stock of a mercantilist's gold and Keynesian unemployment rates have been proposed, and real rates of return and cost-benefit ratios have been implemented. A more recent approach to productivity measurement is the efficiency measure constructed using Data Envelopment Analysis (DEA). This linear programming-based measure has its origin in linear production theory (See Farrell (1957)) but its evolution went down a path somewhat different from economic theory. Our goal is to show the advantages of using microeconomic theory to interpret and apply the DEA model in a different way.

Our intentions are similar to those expressed by Thompson, et al. (1993); that is, "to enhance sound applications of DEA rather than to move the DEA frontier forward." Within this context we explore the advantages of viewing DEA from the perspective of linear production theory. There are some differences between early linear production theory and DEA. Fundamentally linear production theory deals with alternative abstract production processes that could be very different. For example, outputs of one process can be inputs to another. The processes are each assumed to exhibit constant returns to scale, additivity, and irreversibility. The primary questions addressed were the relations among prices and the processes that were chosen, and the existence of competitive equilibria that would dictate the production of an economy with a technology composed of such processes. DEA is primarily concerned with data and only implicitly with the production processes that generate the data. Generally the data are from comparable Decision-Making Units (DMUs) in the same industry. The usual questions addressed concern the efficiency of organizations relative to each other.

Linear production theory is particularly relevant for the interpretation of efficiency measures when data or multipliers are zero in the DEA formulation and solution. This “zeros” problem has caused much discussion in the interpretation of DEA results [see Thompson, et al. (1993)]. It is the basis for concern about the comparability of one DMU to another and for the distinction between declaring a DMU to be efficient or weakly efficient. An additional advantage of using a linear economic model is that the multipliers are comparable (in the same units) across the linear programs that generate the efficient frontier. That is, our multipliers are interpreted as prices of outputs and inputs that may be compared across (DMUs). In contrast, DEA generates multipliers whose units are established by the DMU that is under consideration. Multipliers in the same units help to establish “assurance regions” that eliminate unrealistic values for multipliers, whatever their magnitudes. Schaffnit, Rosen and Paradi (1997) also deal with assurance regions and interpret multipliers as prices. They do so in the context of a traditional output orientation DEA model. In contrast, we use a linear economic model that permits the prices to be measured in the same units so that they are strictly comparable to each other.

In part, these considerations are motivated by a problem and data provided by Shroff (1992). Shroff used DEA as a locational benchmarking tool to support the siting decision for a long-term health care facility in the Northern Virginia region. This work raised several questions about the use of DEA. First, the candidate sites are not clearly the result of any kind of production activity. While each site has both positive and negative attributes, it is not immediately clear that attributes can be interpreted as either inputs or outputs to a production process. Second, each of the 26 sites was characterized by 12 attributes. The usual rule in DEA analysis is to limit the number of inputs and outputs to no more than one-third the number of DMUs. Since the number of sites was limited to the 26 alternatives this rule would have required eliminating at least three of the attributes. The approach of aggregating several inputs to remove zeros from the data and reduce collinearity [See Olesen and Petersen (1996)] is equivalent to requiring two or more implicit multipliers to be equal. This violates the spirit of

DEA. On the other hand, not eliminating some of the attributes leads to results where many of the implicit multipliers are assigned zero values. In addition, several of the sites were characterized by zero values for some of the attributes. Some would argue that these sites were not comparable to the others, but the decision-makers were clearly interested in a comparison of all the sites. Finally, the choice of a site for a long-term health care facility involves both subjective and objective criteria. As Thrall (1995) argues " . . . it is desirable to incorporate user knowledge and preferences rather than for the modeler to act alone." We would argue that if decision-makers cannot include subjective inputs in the analysis, then decision-makers are not likely to use the analysis.

2. Background

Data Envelopment Analysis (DEA) provides a measure of efficiency for one option compared to a set of alternatives. In the DEA tradition, alternatives are called decision-making units (DMUs). Each DMU, is characterized by a vector of outputs and a vector of inputs. Productivity is measured by the ratio of the sum of weighted outputs to the sum of weighted inputs, and DEA assigns weights that are most advantageous to the particular DMU that is under consideration.

The literature on DEA is rich and lengthy. See Charnes, Cooper and Rhodes (1978), Charnes, Cooper, and Thrall (1991) and Thompson, et al. (1993). Some of this literature is focused on the problem of zero values for inputs and outputs and for the implicit multipliers that are calculated by DEA. Kittelsen and Førsund (1992), Banker and Morey (1989), Dryson and Thanassoulis (1988), and Roll, Cook and Golany (1991) explore these subjects. These ideas also form the logic behind the assurance region notion proposed by Thompson, et al. (1986). Thanassoulis and Allen (1998) interpret the constraints that generate the assurance regions not as restrictions on prices but as unobserved DMUs.
The origins of DEA stem from the ratio problem described by Charnes, Cooper and Rhodes (1978). It is

$$\begin{aligned} \max_{u,v} \quad & h_i = u^t y_i / v^t x_i \\ \text{s.t.} \quad & u^t y_j / (v^t x_j) \leq 1 \qquad (j = 1, \ldots, n) \\ & u, v \geq 0 \end{aligned} \tag{1}$$

The vectors u and v are the weights, implicit multipliers, on outputs (y's) and inputs (x's) respectively. The problem is to choose nonnegative values for the multipliers to maximize the ratio of weighted outputs to inputs for one DMU (the ith), subject to the restriction that the ratios for all the DMUs are less than or equal to one. Since the ratio for the ith DMU is also included in the constraints, the value of the objective function is also restricted to be less than or equal to one.

The objective function and each constraint in the ratio model are homogenous of degree zero in the implicit multipliers, that is, if u^* and v^* solve the problem and $a > 0$, then au^* and av^* also solve the problem. This "standard of value" problem is overcome by imposing a normalization constraint on the model; e.g., frequently the input normalization constraint,

$$v^t x_i = 1, \tag{2}$$

is chosen.

The choice of normalization constraint does not affect the value of the objective function or the constraints in problem (1), but it does affect the values of the decision variables (the multipliers). The normalization constraint also permits the ratio problem to be solved as a linear programming problem.

Using the input normalization constraint and letting $Y = \{y_j\}$ and $X = \{x_j\}$, the linear programming representation of the ratio problem is

$$\begin{aligned} \max_{u,v} \quad & u^t y_i \\ \text{s.t.} \quad & v^t x_i = 1 \\ & u^t Y - v^t X \leq 0 \\ & u, v \geq 0. \end{aligned} \qquad (3)$$

This model and its dual form the most basic version of the Data Envelopment Analysis model. This form of the model only restricts the multipliers to be nonnegative but many DEA models add the restriction that the u and v be strictly positive. This version of DEA characterizes DMUs as efficient based on the assumption that all DMUs can be expanded radially, that is, assuming that the underlying production function is characterized by constant returns to scale. Other versions of DEA permit the underlying technology to exhibit variable returns to scale. Thus, DEA can be used to compare efficiency among data generated in a wide class (the isotone class) of functions.

The linear economic model described in this paper **permits** zeros in the multiplier vectors. Our approach to dealing with the zero multiplier problem is similar to the assurance region approach, but it is easier to implement and interpret, and it is more intuitive than other alternatives. In our approach to the problem, the decision-maker can express bounds on multipliers that are all evaluated in the same units (dollars) from one linear program to the next, thus making it easier to make the appropriate value judgment.

3. The Linear Economic Model

There are many linear programming problems that relate to the ratio problem in equation (1). They differ from each other by the choice of the normalization constraint and by the choice of the objective function. One such linear programming problem is the linear economic model presented as equation (4). The objective function for this model is the **difference** between the weighted outputs and the weighted inputs, instead of the output/input ratio. Let w_j be the input/output vector for DMU j and p be the associated vector of implicit multipliers. The elements of w_j are defined so that positive values describe outputs, zeros describe goods that are neither inputs nor outputs, and negative values describe inputs. Let $W = \{w_j\}$. Then the efficiency problem is

$$\begin{aligned} \max_{p} \quad & p^t w_i \\ \text{s.t.} \quad & p^t W \leq 0 \\ & \{\text{normalization constraints}\} \\ & p \geq 0 \end{aligned} \qquad (4)$$

A comparison of one version of the linear economic model, the additive model formulated by Charnes et. al. (1985), and the ratio model is presented in Ahn, Charnes and Cooper (1988), Ali and Seiford (1993), and Frei and Harker (1995). They show that a DMU is efficient under the additive model only if it is efficient under the ratio model. The additive model has an unusual normalization constraint that requires each of the multipliers to be greater than or equal to one. That is

$$\begin{aligned}&\max_{p} \quad p^t w_i\\ &\text{s.t.} \quad p^t W \le 0\\ &\qquad -p^t I \le -e \\ &\qquad p \ge 0,\end{aligned} \tag{5}$$

where e is a column vector of ones. These constraints rule out free goods. They are too restrictive and unnecessary for defining a numeraire for the linear economic model. The additive model constraints cause its dual (the envelopment model) to maximize the sum of the dual slack variables. That is

$$\begin{aligned}&\min_{\lambda, S} \; -e^t S\\ &\text{s.t.} \quad W\lambda - S = w_i\\ &\qquad \lambda, S \ge 0\end{aligned} \tag{6}$$

This rather unusual objective function is merely the result of the normalization constraint. Unlike the ratio model, a normalization constraint is not required to transform the linear economic model into a linear programming problem. But, since the maximum value of the objective function is zero and since the trivial solution of p = 0 is feasible, then some normalization constraint is required for the optimization problem described by equation (4).

For example, consider the normalization constraint that confines the price vector to the unit simplex,

$$p^t e = 1. \tag{7}$$

Then the dual to the linear economic model at (4) is

$$\begin{aligned}&\min_{\lambda,\theta} \; \theta\\ &\text{s.t.} \quad W\lambda + \theta e \ge w_i\\ &\lambda \ge 0, \; \theta \text{ unconstrained}\end{aligned} \tag{8}$$

The model described by equation (8) is much closer to the usual envelopment problem. It is easy to show that an input-output vector is efficient under problem (3) if and only if it is efficient under problem (4) with this normalization constraint.

If the input output vector (x_i, y_i) is efficient under problem (3) then

$u^{*t}y_i = 1,$
$u^{*t}y_i - v^{*t}x_i = 0,$
and $u^{*t}Y - v^{*t}X \le 0.$
Let $q^t = (u^t, v^t) / (u^t, v^t)e$
So that $q^t e = 1$
Then $q^t w_i = (u^{*t} y_i - v^{*t}x_i) / (u^t, v^t)e = 0$
and $q^t W = (u^{*t}Y - v^{*t} X) / (u^t, v^t)e \le 0.$

So q is an optimal solution to (4) and (x_i, y_i) is efficient by (4).
Likewise let p^* be an optimal solution to (4) where w_i is efficient. Then

$p^{*t}w_i = 0,$
$p^{*t}e = 1,$
and $p^{*t}W \leq 0.$
Let $(r^t, s^t) = p^{*t} / (\mathbf{0}^t, x_i^t)\, p^*$
so that $s^t x_i = 1.$
Then $(r^t y_i - s^t x_i)\ (\mathbf{0}^t, x_i^t)\, p^* = 0$
so $r^t y_i - s^t x_i = 0$
and $r^t y_i = 1.$
Likewise $(r^t, s^t)\ (X\text{-}Y)\ (\mathbf{0}^t, x_i^t)\, p^* \leq 0$
so $r^t X - s^t Y \leq 0.$

Hence r and s are optimal solutions to (3) and (x_i, y_i) is efficient at that solution.

The problem described by equation (4) is familiar to economists. If the multipliers are interpreted as prices, then solution algorithm searches for the set of prices (for outputs and inputs) that make DMU_i as profitable as possible. Or, put the other way around, given those prices, if DMU_i views W as a technology matrix, then its input output vector is the choice of technology to maximize profits if DMU_i is efficient. The constraints on the problem require that the prices be consistent with a competitive equilibrium (no DMU makes economic profits). Alternative normalization constraints define different numeraires for the general equilibrium. While there is more to describing a general equilibrium solution for a linear economy all of this is consistent with that solution. This story is well told by Dorfman, Samuelson, and Solow (1958), Gale (1960), and Karlin (1959) among others.

Any efficient DMU will have a maximum economic profit of zero in equation (4), and at the associated equilibrium set of prices, that DMU is portrayed as a viable competitive firm. If a DMU is not efficient, then there exists no set of equilibrium prices at which the DMU would be a viable competitive firm in the presence of the others. Furthermore, the set of prices calculated when an inefficient DMU is evaluated gives its minimum loss in its most favorable competitive equilibrium.

When the same normalization constraint is used for all of the programs in the problem described by equation (4) then all of the prices are comparable across the linear programs. The choice of a single good as a numeraire, for example $p_1 = 1$, has the advantage of making the prices from all of the linear programming problems comparable. This gives the linear economic model a unique advantage in forming assurance regions because the decision-maker can easily incorporate the assurance region bounds as additional constraints on the problems. Choosing good one as numeraire does preclude weak efficiency with respect to good one though.

In addition to the interpretation of efficient DMUs as viable firms in some competitive equilibrium, the perspective of linear economic theory provides another interpretation of the DEA. To model variable returns to scale the dual variables to the problem at (3) are constrained to add to one. This requires an extra row of ones in the Y matrix of the problem described by equation (3). The implicit multiplier associated with that row is also included in the objective function and is unconstrained in sign. Thus, from the perspective of the linear economic model, variable returns to scale is modeled as if there were an unobserved input (if the multiplier is negative) or output (if the multiplier is positive) for each of the firms whose value is one. For example, we might imagine a service, managerial talent, whose value is one for all firms. When the implicit multiplier for that service is positive then managerial talent is an output of the firms that gives rise to decreasing returns to scale (increasing all of the other inputs and outputs results in no change to this output). When the implicit multiplier is negative then managerial talent is a negative output (an input) that gives rise to increasing returns to scale.

4. An Application of the Linear Economic Model

A study was initiated and funded by INOVA Health Systems, the primary long-term health care provider in the Northern Virginia region to identify sites for locating a facility. Based on the results of the model and other managerial considerations, a site was selected and a facility was constructed. We revisit the problem and compare the linear economic model and a more traditional DEA model of the site selection problem.

INOVA's proposal to build a new long-term care facility in Northern Virginia was based on increased demand for long-term care and the relative cost advantage of a long-term care facility to a hospital. INOVA's decision to build a long-term care facility was influenced by health regulatory and zoning requirements, as well as economic and political considerations. Other important influences were INOVA's not-for-profit status, regulations regarding bed-type and acuity levels of care, and regulations with respect to pay-type (Medicare or private).

INOVA's planning staff proposed that a 120-bed facility be constructed and subjected to several additional managerially imposed constraints. One constraint imposed by INOVA was that the pay-type in the new facility be evenly distributed between private-pay and government-pay patients. If the only criterion were profit maximization, then private-pay patients would be preferred, since INOVA incurs a monetary loss on each government-pay bed. But, INOVA's not-for-profit status and its sense of social responsibility made such an allocation unacceptable. An equal payment-type split was chosen and imposed as a constraint.

Restrictions on the bed types allocated to different acuity care levels were also imposed as:

Category I - Subacute care patients. The patients in this category include symptomatic HIVs, head and spinal chord injuries, and ventilator dependencies. INOVA's management allocated twelve beds to this category.

Category II - Skilled care patients. The total number of beds allocated by INOVA's management to this category was 36.

Category III - Intermediate care patients. These are elderly patients needing assistance, or patients who have recently suffered a stroke or post cardiovascular arrest. The total number of beds allocated by INOVA's management to this category was 72.

Thus, the mix of bed-types and pay-types were both externally imposed. The analysis team was asked by INOVA's management to address the following question: Given all relevant variables, tell us the preferred area within Northern Virginia for building a new long-term care facility. Shroff (1992) analyzed this problem in detail, using the DEA methodology as originally proposed by Charnes, Cooper, and Rhodes (1978). The problem falls within the realm of multiple criteria [See Belton (1992)], location benchmarking[See Kao and Yang (1992)]. The details of the study are contained in Shroff (1992) and summarized in Shroff, et al. (1998).

DMUs were defined as regional planning districts. That is, the Northern Virginia region was divided into a set of demand points using a strategy similar to that used by Plane and Hendrick (1977) for siting fire stations within the city of Denver. This resulted in 26 DMUs, each being a planning district within the Northern Virginia region.

The inputs and outputs described below differ from those used in the original study. We have structured the problem to take advantage of additional information that could not be included in a straightforward application of the ratio problem. This new formulation results in a data matrix that contains many zeros, an interesting by-product for the results of this analysis. The original study contained all positive values. We used the following inputs and outputs:

w_1 = Projected subacute care bed demand, measured in the number of potential private patients (up to 6) for the 6 subacute care beds in the 120 bed facility.

w_2 = Projected skilled care bed demand, measured in the number of potential private patients (up to 18) for the 18 skilled care beds in the 120 bed facility.

w_3 = Projected intermediate care bed demand, measured in the number of potential private patients (up to 36) for the 36 intermediate care beds in the 120 bed facility.

w_4 = Projected extra bed demand. This variable is demand for private beds in the facility that is in addition to capacity for particular care (that is, in addition to capacity for sub-acute, skilled, and intermediate care) but within the constraints of the 60 private beds. For example, if 19 private skilled care patients were projected and the sum of w_1 through w_3 were less than 60, then w_4 would be one.

w_5 = Projected excess bed demand. This variable is private demand for beds in excess of the 60 private bed constraint. It is valuable only to the extent that it helps to ensure that the private portion of the facility is continuously full.

w_6 = Desirability of Location, a 0-1 variable assessed by survey methods. This variable related mainly to accessibility by major transportation networks. A value of one indicates that this DMU is near major networks.

w_7 = Land acquisition costs. This variable was measured in thousands of dollars from parcels of land that met zoning and regulatory requirements.

w_8 = Projected demand for professional staff (i.e., registered nurses and licensed practitioner nurses) in the planning district.

w_9 = Projected demand for other professional staff (i.e., dietitians and therapists) in the planning district.

w_{10} = Projected demand for semiskilled staff (i.e., certified nurses, nurses aides, and orderlies) in the planning district.

w_{11} = Projected demand for physicians (i.e., practitioners, internists, primary care physicians, and psychiatrists).

w_{12} = Facility construction. This data value is negative one for all DMUs (i.e., a new facility will be constructed whatever the choice of location).

To appropriately consider competitive pressures within the planning districts, inputs were measured on a per-bed basis when appropriate. For example, five doctors available to service 500 beds are much different from five doctors available to service 100 beds. To avoid this problem, the demands for each of the caregivers, w_8 through w_{11} are reported as the total (INOVA and all competitors) number of long-term care beds divided by the number of caregivers in the district. All of the inputs (w_7 through w_{12}) are entered as negative values.

Given these inputs and outputs, we can think of the implicit multipliers of the linear programming problems as determining input and output values that make each site look most favorable in a breakeven analysis. Of course these favorable indications only make sense if the values that the linear programs calculate are reasonable in the minds of the decision-makers.
In many instances sites have zero values for an output or an input. Some have argued that the various DMUs compared by DEA should be "similar." They also argue that DMUs that do not produce a product when others do are dissimilar, and that they should not be compared by

DEA. Our point of view is in dramatic contrast. We believe it is the decision-making situation that dictates the DMUs to be compared and that it is the analyst's task to design methods to compare the alternatives. Our approach to evaluating the alternative sites can deal with such "dissimilar" DMUs. Others have argued for the approach of aggregating several inputs to remove zeros from the data set and to reduce collinearity [See Olesen and Petersen (1996)]. This approach requires that two or more inputs be valued equally. This violates the spirit of DEA and in any case is unnecessary with our approach.

5. The Output of the Linear Economic Model

For our first analysis, mainly for comparison purposes, we solve the problem in equation (4) with only an upper bound constraint on the prices. The DMUs identified as efficient by this analysis would also be identified as efficient by the ratio problem; however, the values and interpretation of the decision variables are different. Land acquisition cost in thousands of dollars, w_7, was used as the numeraire; thus, all implicit multipliers are in units of thousands of dollars. The results of this analysis are presented in Table 1.

The prices in Table 1 illustrate the problem of using DEA without any restrictions on the multipliers; i.e., seventeen of the twenty-six planning districts are declared to be efficient choices for the long-term care facility. Thus, without restrictions, DEA provides little guidance for locating the facility. Furthermore, every planning district has a zero multiplier for at least one attribute. In every case at least one attribute can be regarded as unimportant in describing the efficient district. The ratio model with popular normalization rules would assign the same efficient sites an efficiency score of one. Although the relative scores of the other districts would be different, none of them would be chosen as efficient by the ratio model. These results should be tempered with two observations. First, it is often true that the situations that generate zero prices correspond to districts that have a zero value for the corresponding attribute. Second, all of the results are characterized by alternate optima, so wide ranges of implicit multipliers correspond with efficient solutions for many planning districts.

The advantage of the linear economic model in understanding this situation is the ability to interpret the implicit multipliers as prices. Because of our numeraire choice, prices are in thousands of dollars. Therefore the prices that show McLean to be an efficient site have the "location desirability" variable evaluated at \$264,300 but "construction cost" at 0. Clearly a site that must have free construction to be efficient has little to recommend it.

The linear economic model allows the decision-maker to use knowledge of relative values to impose relevant constraints on the multiplier vector. To illustrate this we impose constraints on the multipliers as follows:

$$\begin{aligned} 150 &\le P_i \le 250, && i = 1, \ldots, 4; \\ 10 &\le P_i \le 100, && i = 5, 6, 8, 9, 10; \\ 10 &\le P_{11} \le 500; && \\ 10{,}000 &\le P_{12} \le 50{,}000. && \end{aligned} \qquad (9)$$

That is, the present value of the stream of net benefits coming to the facility from one of the patient beds is between \$150,000 and \$250,000. The solutions to the new programs are presented in Table 2. Even these very liberal restrictions on the values of site attributes dramatically reduce the number of efficient planning districts from seventeen to six.

The large variation of output prices within each district in Table 2 is of concern. For example, Potomac prices w_4, extra bed demand, at \$250,000 per bed; much higher than the demand for other beds. This is the case even though extra bed demand is clearly less valuable than the other outputs. To investigate the impact of restricting this variance, we further constrain the programs so that

$$P_1 = P_2 = P_3 = P_4. \quad (10)$$

This has the effect of aggregating the four measures of output into one.

These additional restrictions generate the results in Table 3, where the number of efficient districts is reduced to two: Pohick and Central Arlington. In comparing these two districts, high prices for inputs and outputs seem to favor Pohick, while lower prices seem to favor Central Arlington. The single exception to this rule is the desirability of location attribute, since it must receive a low value, $10,000, for Pohick to be favored. Even with these restrictions the solution is characterized by alternate optima. Here, Central Arlington is still efficient when evaluated at the prices that are most favorable to Pohick. Ultimately, as reported by Shroff et al. (1998), a site in Pohick was selected based on criteria in this study as well as other considerations.

The point of this paper is not that these are the correct restrictions to impose on the problem, nor is the point that the choice of a site should be based only on the inputs and outputs that are included in the linear model. Instead, we believe that formulating the choice problem so that the decision-maker's beliefs about ranges of resource values can be easily included in the analysis will result in higher quality decisions than otherwise. This point was noted and discussed in detail by Thompson, et al. (1993). The linear economic model is such a formulation.

An additional advantage of the linear economic model was not explored in this application. Consider a site attribute that is not clearly an advantage or disadvantage of a site; for example, proximity to a major highway. For some decision-makers the highway may be a positive that permits easy access to the site, and for others it may be viewed as a source of noise and air pollution that detracts from patient care. If we relax the requirement that all of the prices be nonnegative, then we can permit proximity to be either a positive or negative attribute. This permits the decision-makers to weigh in with their beliefs, if they are necessary to make a choice. In other contexts, where some of the data are observations of noxious outputs or observations of the consumption of commonly held inputs, then allowing negative prices would allow us to evaluate efficiency with respect to pollution and productive output as well. Finally we note that if we want to model variable returns to scale then we need only add another row to the input-output vector for each alternative and let its price be unconstrained in sign as well.

6. Summary

Data Envelopment Analysis (DEA) is a useful way to analyze data on productive activities. It provides a consistent comparison of one activity to others in circumstances that are particularly favorable to the activity that is being examined. This paper proposes looking at DEA as an attempt to construct a set of general equilibrium prices for a linear economy. The examination of DEA from this point of view provides a strong rationale for permitting zeros in the data vectors and in the vector of implicit multipliers. This approach also yields prices (i.e., multipliers) from all of the linear programming problems that are comparable, thus simplifying the use of judgment in evaluating efficient DMUs. It also suggests ways in which DEA can be expanded to include the analysis of activities where goods are not consistently inputs or outputs and where some outputs may be noxious byproducts. Some of these ideas are illustrated with a siting decision for a long-term health care facility.

7. References

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Table 1. - Efficiency Scores and Implicit Multipliers, Unrestricted.

Planning District	Eff.	Score	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
Annandale	YES	0.0	0.0	1244.0	0.0	4.3	0.0	0.0	1.0	0.0	0.0	4.4	9.3	22240.7
Bailey's	YES	0.0	0.0	1244.6	0.0	4.4	0.0	0.0	1.0	0.0	0.0	4.4	9.9	22240.7
Bull Run	YES	0.0	0.0	1208.4	17.0	23.4	0.0	0.0	1.0	0.0	6.2	0.0	13.4	22240.7
Fairfax		-11.0	0.0	1242.6	0.0	0.0	0.0	43.0	1.0	0.0	0.0	0.0	0.0	22239.5
Jefferson		-15.8	0.0	1241.9	0.3	0.0	0.0	49.0	1.0	0.0	0.0	0.0	0.0	22239.5
Lincolnia		-109.6	0.0	7.1	0.0	0.0	0.0	43.0	1.0	0.0	0.0	0.0	0.0	0.0
L. Potomac		-125.4	0.0	7.1	0.0	0.0	0.0	43.0	1.0	0.0	0.0	0.0	0.0	0.0
McLean	YES	0.0	58.6	29.4	2.6	0.2	0.0	264.3	1.0	0.0	49.2	0.0	131.8	0.0
Mt. Vernon	YES	0.0	22.4	11.7	0.0	2.1	0.0	78.0	1.0	0.0	19.2	0.0	8.3	0.0
Pohick	YES	0.0	22.6	25.4	0.0	0.0	0.0	88.7	1.0	16.9	19.5	0.0	34.5	135.3
Rose Hill	YES	0.0	22.6	25.4	0.0	0.0	0.0	88.7	1.0	16.9	19.5	0.0	34.5	135.3
Springfield	YES	0.0	18.9	26.2	0.2	0.0	0.1	87.4	1.0	16.9	19.2	0.5	38.6	135.3
U. Potomac	YES	0.0	94.2	7.4	0.0	3.2	0.0	0.0	1.0	9.8	17.7	0.0	0.0	135.3
Vienna	YES	0.0	372.2	0.0	0.0	43.2	2.4	0.0	1.0	0.0	6.0	0.0	114.8	959.6
Southwest Arlington		-32.9	1184.7	0.0	1193.8	1194.2	0.0	46.6	1.0	0.0	0.0	0.0	0.0	50000.0
Central Arlington	YES	0.0	1179.0	0.0	1193.7	1194.3	0.6	53.4	1.0	0.0	0.0	0.0	0.0	50000.0
North Arlington		-6.3	0.0	0.0	1234.3	1285.7	0.0	0.0	1.0	145.2	0.0	0.0	0.0	50000.0
Southeast Arlington		-34.0	0.0	2776.4	0.0	0.0	0.0	43.0	1.0	0.0	0.0	0.0	0.0	49847.6
Seminary	YES	0.0	0.0	2586.1	84.9	89.2	0.0	24.6	1.0	1.0	1.2	2.3	1.2	49850.8
Potomac	YES	0.0	39.9	2603.2	74.2	78.9	1.6	4.6	1.0	0.0	0.0	1.0	0.0	49849.8
E. Alexandria	YES	0.0	0.0	2775.5	0.4	0.1	0.0	49.1	1.0	0.0	0.0	0.0	0.0	49847.6
Central Loudon		-89.5	0.0	2776.1	0.0	2.5	0.0	0.0	1.0	0.0	0.0	4.8	0.0	49847.6
North Loudon		118.0	0.0	2776.4	0.0	0.0	0.0	24.5	1.0	0.0	0.0	0.0	0.0	49847.6
East Prince William	YES	0.0	0.0	2766.4	0.0	0.0	0.0	24.5	1.0	0.0	0.0	0.0	0.0	49847.6
Central Prince William	YES	0.0	0.0	2776.4	0.0	0.0	0.0	24.5	1.0	0.0	0.0	0.0	0.0	49847.6
North Prince William	YES	0.0	0.0	2776.4	0.0	0.0	0.0	24.5	1.0	0.0	0.0	0.0	0.0	49847.6

Table 2. - Efficiency Scores and Implicit Multipliers, Restricted.

Planning District	Eff.	Score	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
Annandale		-2499.3	150	173	150	213	10	10	1	10	10	10	100	10000
Bailey's	YES	0.0	151	154	172	232	10	78.2	1	10	10	10	234	10000
Bull Run		-581.3	150	150	190	250	10	10	1	100	13	10	100	11016
Fairfax		-2428.6	150	173	150	213	10	11.4	1	10	10	10	100	10000
Jefferson		-2953.0	150	173	150	213	10	11.4	1	10	10	10	100	10000
Lincoln		-8112.2	150	182	150	150	10	10	1	10	10	10	100	10000
L. Potomac		-9420.7	150	182	150	150	10	10	1	10	10	10	100	10000
McLean	YES	0.0	153	191	150	250	17	40.2	1	10	10	10	401	10000
Mt. Vernon		-38.3	250	150	150	227	11	10	1	100	10	10	100	10052
Pohick	YES	0.0	250	150	150	234	12	10	1	100	32	18	100	10000
Rose Hill		-503.7	150	150	250	250	10	100	1	100	100	100	296	11654
Springfield		-2028.9	150	250	187	150	10	10	1	100	100	100	500	10000
U. Potomac		-319.7	250	195	150	191	10	10	1	100	100	10	100	10002
Vienna	YES	0.0	250	150	150	238	10	10.9	1	99.6	44	10	100	10003
Southwest Arlington		-568.3	250	153	150	219	10	10	1	10	10	10	100	10002
Central Arlington	YES	0.0	250	153	150	219	10	10	1	10	10	10	100	10002
North Arlington		-221.9	150	150	164	250	10	10	1	10	10	10	100	10642
Southeast Arlington		-6278.2	150	173	150	213	10	10.1	1	10	10	10	100	10000
Seminary		-10.5	150	158	150	250	10	100	1	10	48	100	106	10000
Potomac	YES	0.0	150	158	150	250	10	100	1	10	48	100	106	10000
E. Alexandria		-311.3	150	150	237	250	10	72.5	1	10	10	100	500	10985
Central Loudon		-2522.2	150	178	150	215	10	10	1	10	10	51	100	10000
North Loudon		-3113.9	150	193	150	150	10	10	1	10	10	100	100	10000
East Prince William		-329.8	150	150	237	250	10	10	1	100	100	100	100	11923
Central Prince William		-852.4	150	150	237	250	10	10	1	100	100	100	100	11923
North Prince William		-2851.0	150	173	150	213	10	10	1	10	10	10	100	10000

Table 3. - Efficiency Scores and Implicit Multipliers, $P_1 = P_2 = P_3 = P_4$.

Planning District	Eff.	Score	P_1 - P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}
Annandale		-3584.0	159.5	10	10	1	10	10	10	100	10000
Bailey's		-389.8	187.2	10	10	1	10	10	10	500	10000
Bull Run		-1111.9	174.5	10	10	1	100	100	100	100	10000
Fairfax		-3656.6	158.0	10	100	1	10	10	10	100	10000
Jefferson		-3591.5	158.0	10	100	1	10	10	10	100	10000
Lincolnia		-8186.2	159.5	10	10	1	10	10	10	100	10000
L. Potomac		-9512.4	159.5	10	10	1	10	10	10	100	10000
McLean		-88.8	197.2	10	100	1	100	100	10.5	500	10000
Mt. Vernon		-1207.4	250.0	10	10	1	10	10	10	100	15430
Pohick	YES	0.0	250.0	10	10	1	100	100	100	315.5	13632
Rose Hill		-702.6	250.0	10	10.1	1	100	100	100	315.5	13632
Springfield		-2437.7	196.5	10	10	1	100	80.2	10	500	10000
U. Potomac		-505.2	174.5	10	10	1	100	100	100	100	10000
Vienna		-1131.5	250.0	10	10	1	10	10	10	500	13766
Southwest Arlington		-1667.2	158.0	10	100	1	10	10	10	100	10000
Central Arlington	YES	0.0	158.0	10	100	1	10	10	10	100	10000
North Arlington		-1717.2	158.0	10	100	1	10	10	10	100	10000
Southeast Arlington		-6698.6	158.0	10	100	1	10	10	10	100	10000
Seminary		-1665.1	158.0	10	100	1	10	10	10	100	10000
Potomac		-1628.1	158.0	10	100	1	10	10	10	100	10000
E. Alexandria		-516.4	250.0	10	10	1	10	10	100	500	13555
Central Loudon		-3646.7	159.5	10	10	1	10	10	10	100	10000
North Loudon		-3246.0	163.0	10	10	1	10	10	100	100	10000
East Prince William		-504.1	174.5	10	10	1	100	100	100	100	10000
Central Prince William		-1032.6	174.5	10	10	1	100	100	100	100	10000
North Prince William		-4115.7	159.5	10	10	1	10	10	10	100	10000