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Metaheuristic for the Traveling Salesman Problem"**

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The aim of this technical note is to provide some comments on the paper Tsubakitani and Evans (1998) concerning a "jump search" metaheuristic for the traveling salesman problem (TSP) and to disclose the origin and nature of certain recent developments that yield promising outcomes for the TSP. Due to the fact that the T & E paper had already been accepted in July 1996, some innovations in the TSP area had occurred before its publication. Consequently, through no fault of the authors, their work gives a somewhat misleading impression about the current state of the art on metaheuristics and in particular about the methods that provide the best results for the traveling salesman problem.

The T & E paper presents concepts that must unfortunately be regarded primitive in relation to those currently applied in local search strategies. The authors base their conclusions on computational tests that compare their outcomes to previous results that have by far been superseded by superior findings.

The paper additionally contains an oversight, which stems from the authors' portrayal of contrasts between their re-starting ideas and certain ideas of tabu search (TS), apparently without being aware that tabu search includes related (and more advanced) ideas. A variety of re-starting strategies are available within the TS framework, which benefit from the same adaptive memory processes and supporting intensification and diversification procedures that underly TS implementations during subsequent phases of search. (See, for example, the book Glover and Laguna, 1997.)

From a computational standpoint, a comparison with recent outcomes discloses that the test results of Tsubakitani and Evans are significantly dominated by those reported in the current TSP literature, and notably by those of Rego (1998) reported in the EJOR Special Issue on Tabu Search. T & E reasonably claim that it is difficult to compare their results with those obtained by other alternative algorithms in the literature -- because the results typically reported are either the average of multiple runs or the best among them -- but such a claim does not prove relevant in this instance. At the same time, the supposed lack of strategy in starting phases of TS methods is also clearly belied by this work. The P-SEC and F-SEC algorithms described in the EJOR study by Rego generate starting solutions systematically according to a well defined rule that integrates a simple TS approach with ejection chain procedures. (Ejection chain procedures likewise originated within the tabu

search setting.) In this sense, the number of jumps reported by the "jump search" algorithms corresponds to the number of runs used by P-SEC and F-SEC algorithms. This permits accurate comparisons between the methods by adding the CPU time for each run.

The difficulty that may exist in comparing the jump search results with those of present algorithms derives from the fact that current TSP research no longer commonly devotes attention to problems as small as those in the T & E study. Thus, for example, the TS/ejection chain study publishes results only for two of the problems tested with jump search (among 67 problem instances that include problems containing up to 7397 nodes). The reason for disregarding smaller test problems is that the heuristic procedures available today usually find optimal solutions for problems of relatively small sizes in a very few seconds. This is illustrated in the table below, which provides comparative results between the jump search based algorithms and those provided by the alternative framework indicated. Specifically, columns 3-8 report results from Tsubakitani and Evans (1998) for their six algorithm variations of three metaheuristic approaches: jump search, iterative local search with random re-start, and tabu search, each of which using 3-opt and 2-opt neighborhood search procedures. The last two columns report results produced (to the present study) by the two versions of the tabu search algorithm based on a subpath ejection chain method described in Rego (1998). All these algorithms are written in C, compiled with a Borland C++, and were run on a 386/33Mhz based computer. Also, as P-SEC and F-SEC algorithms find the optimal solution for all problem tested the CPU times for these algorithms always correspond to the time necessary for this solution to be reached (which is also a stopping criterion used in the T&E algorithms). These similarities make the solution times roughly comparable which gives a special relevance for the comments discussed in this note.

PROBLEM	OPT. SOL.	JMP3 OPT	RND30 PT	TAB30 PT	JMP20 PT	RND20 PT	TAB20 PT	P-SEC	F-SEC
33-city									
Tour Cost	1086	10861	10861	10861	10861	10861	10861	1086	1086
% Abv. Opt.	1	0.00	0.00	0.00	0.00	0.00	0.00	1	1
CPU Sec.		1.8	12.8	15.9	0.93	3.1	0.7	0.00	0.00
# of Jumps		1	2		14	23	1	0.3	0.3
								1	1
42-city									
Tour Cost		699	699	699	699	701	699		
% Abv. Opt.	699	0.00	0.00	0.00	0.00	0.29	0.00	699	699
CPU Sec.		8.6	140.4	419.6	2.0	14.1	89.2	0.00	0.00
# of Jumps		4	8		4	23		4.2	6.1
								1	1
50-city									
Tour Cost		426	428	426	435	435	426		
% Abv. Opt.	425	0.24	0.71	0.24	2.35	2.35	0.24	425	425
CPU Sec.		714.9	303.2	1258.7	7.1	28.2	215.9	0.00	0.00
# of Jumps		4	8		4	23		3.28	7.8
								1	1
57-city									
Tour Cost		12955	12955	12985	13056	13076	13133		
% Abv. Opt.	1295	0.00	0.00	0.23	0.78	0.93	1.37	1295	1295
CPU Sec.	5	1064.0	571.39	2169.1	10.3	47.6	367.2	5	5
# of Jumps		93	9		105	64		0.00	0.00
								5.7	18.9
								2	1
75-city									
Tour Cost		537	541	536	546	543	537		
% Abv. Opt.		0.37	1.12	0.19	2.06	1.50	0.37		
CPU Sec.	535	5889.8	1602.3	7200.0	28.4	142.1	1650.8	535	535
# of Jumps		156	10		156	88		0.00	0.00
								7.8	6.3
								1	1
100-city									
Tour Cost		21282	21292	21573	21320	21602	21360		
% Abv. Opt.		0.00	0.05	1.37	0.18	1.50	0.37		
CPU Sec.	2128	333.61	4960.0	7200.0	92.3	446.4	6740.1	2128	2128
# of Jumps	2	5	8		191	96		2	2
								0.00	0.00
								5.8	6.2
105 city									
Tour Cost		14379	14379	14479	14548	14477	14379		
% Abv. Opt.		0.00	0.00	0.70	1.18	0.68	0.00		
CPU Sec.		111.50	5927.7	7200.0	89.5	542.3	727.4		
# of Jumps	1437	1	7		197	99		1437	1437
	9							9	9
Avg % Abv Opt.		0.09	0.26	0.39	0.94	1.36	0.33	0.00	0.00
								5.0	8.3
								1	1
								0.00	0.00

33-city - Karg and Thonpson (1964)
42-city - Dantzig et al., (1954)
50-city - Christofides and Eilon (1969)
57 city - Karg and Thonpson (1964)
75-city - Christofides and Eilon (1969)
100-city - Krolak and Marble (1971)
105 city - Lin and Kernighan (1973)

As the table shows, both of the TS/ejection chain algorithms significantly outperform all the other algorithms. (As shown in the more extensive study of Rego (1998), the F-SEC algorithm also notably outperforms the best current alternative methods in the literature, such as enhanced versions of the Lin-Kernighan approach, except for 'iterated' methods that are allowed to consume massively greater amounts of computation time.) Since the jump search approach relies on 2-opt and 3-opt moves, we should remark that tabu search implementations based on such moves exist in the literature whose results are also significantly superior to those provided by jump search. (See, e.g., Zachariasen and Dam, 1996, whose implementations likewise address much larger problems than examined with the jump search approach.)

In addition to its greater computational efficiency, the tabu search guidance scheme reported here appears to be much more robust than the jump search scheme. For example, the JMP3OPT procedure takes about 333 seconds to find the optimal solution for the 100-city and in addition takes more than 17 times this time to find a (suboptimal) solution for a smaller problem with 75 cities ! (The latter time is more than three orders of magnitude slower than taken by the TS approach to obtain a solution of the same quality.) Also, it may be noted that TAB3OPT and JMP3OPT usually find solutions of similar quality, whereas, by contrast, TAB2OPT performs significantly better than JMP2OPT (which fails drastically for the 50-city, 75-city and 105-city problems).

Finally, we point out that the strategic framework of Rego (1998) has also been effectively applied to other settings such as vehicle routing (Rego, 1998a; Rego and Roucairol, 1996) and crew scheduling (Cavique, Rego and Themido, 1998).

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