Randomized Colorings of Graphs

by

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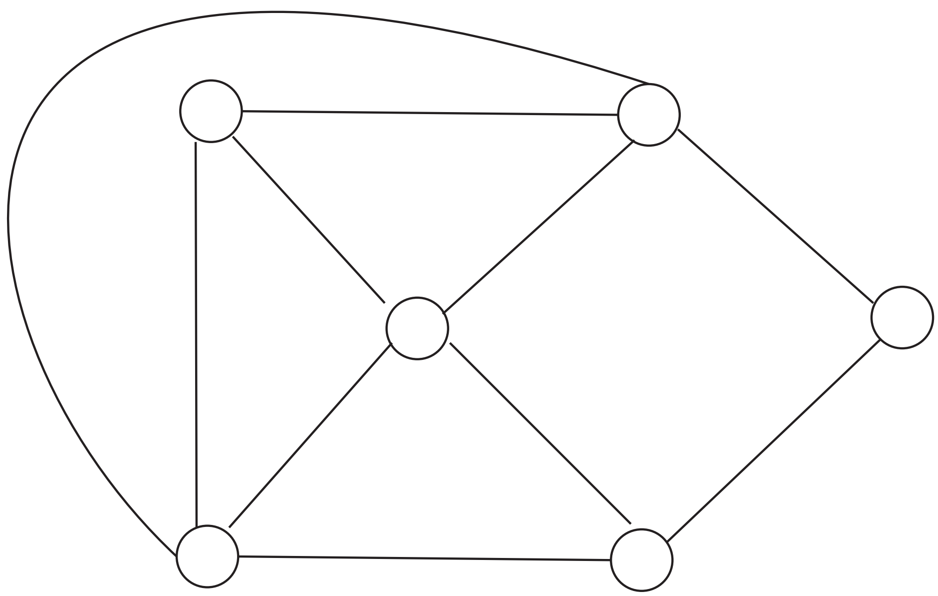
Abstract:

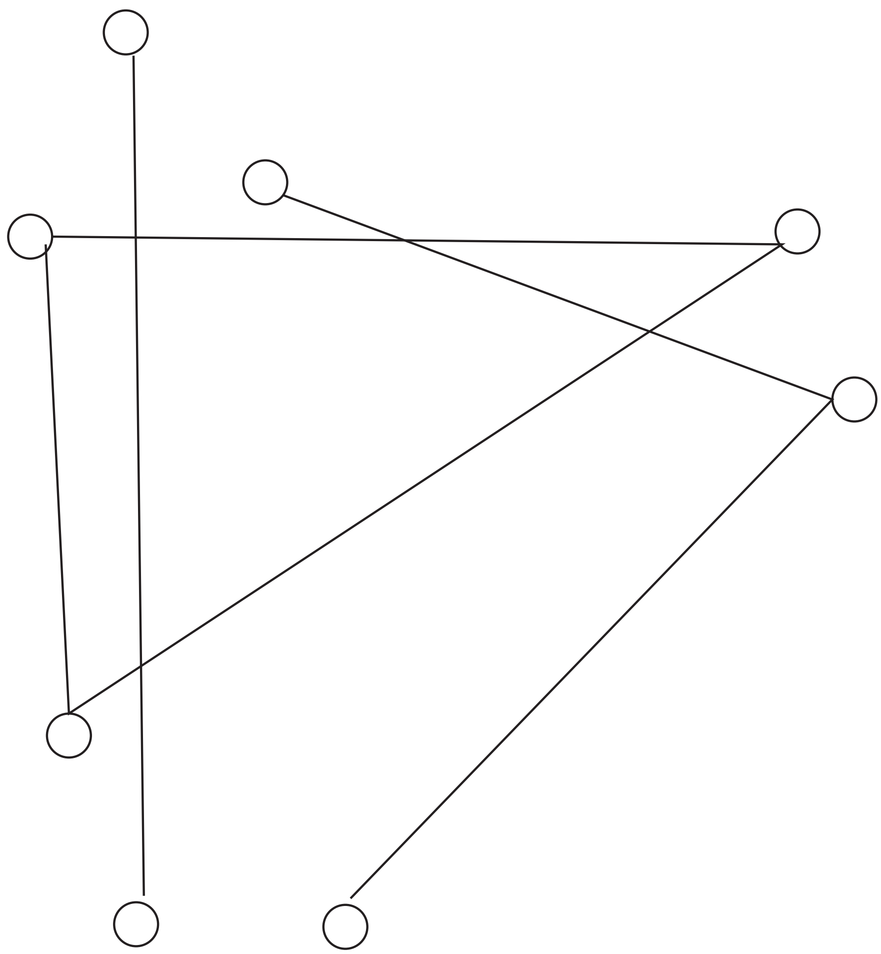
We provide an efficient randomized algorithm for 3-coloring a graph. The problem of 3-coloring a graph is NP-Complete. Heuristics for solving NP-Complete problems are important because many important assignment problems are NP-Hard.

A 4-colorable graph



Adjacent Vertices
Receive Different Colors
4 - Colorable

A 3-colorable graph



The best known deterministic polynomial time algorithm (polynomial in the number of vertices of the graph) for coloring a graph uses $n^{\frac{3}{8}}$ colors.

Pratical assisgnment problems often involve 100,000 more nodes or more that must be considered.

A. D. Petford and D. J. A. Welsh A randomised 3-colouring algorithm. Graph colouring and variations. Discrete Math. 74 (1989), no. 1-2, 253–261

Petford and Welsh provide a method for quickly 3−coloring a graph.

This method requires successive passes through the vertex set.

Pass 1. Randomly color the graph with three colors. (There are many color clashes present)

Successive passes. For each vertex, choose red as its color with probability proportional to $2^{-\text{ number of red neighbors}}$.

Normalize so the sum of the probabilities is one:

Prob (red) + Prob (blue) + Prob (green) =1

$$\text{Prob (red)} = \frac{2^{-r}}{2^{-r}+2^{-b}+2^{-g}}$$

$$\text{Prob (blue)} = \frac{2^{-b}}{2^{-r}+2^{-b}+2^{-g}}$$

$$\text{Prob (green)} = \frac{2^{-g}}{2^{-r}+2^{-b}+2^{-g}}$$

Example red = 3 , blue =4, green =4

$\text{Prob (red)} = \frac{1}{2}$

$\text{Prob (blue)} = \frac{1}{4}$

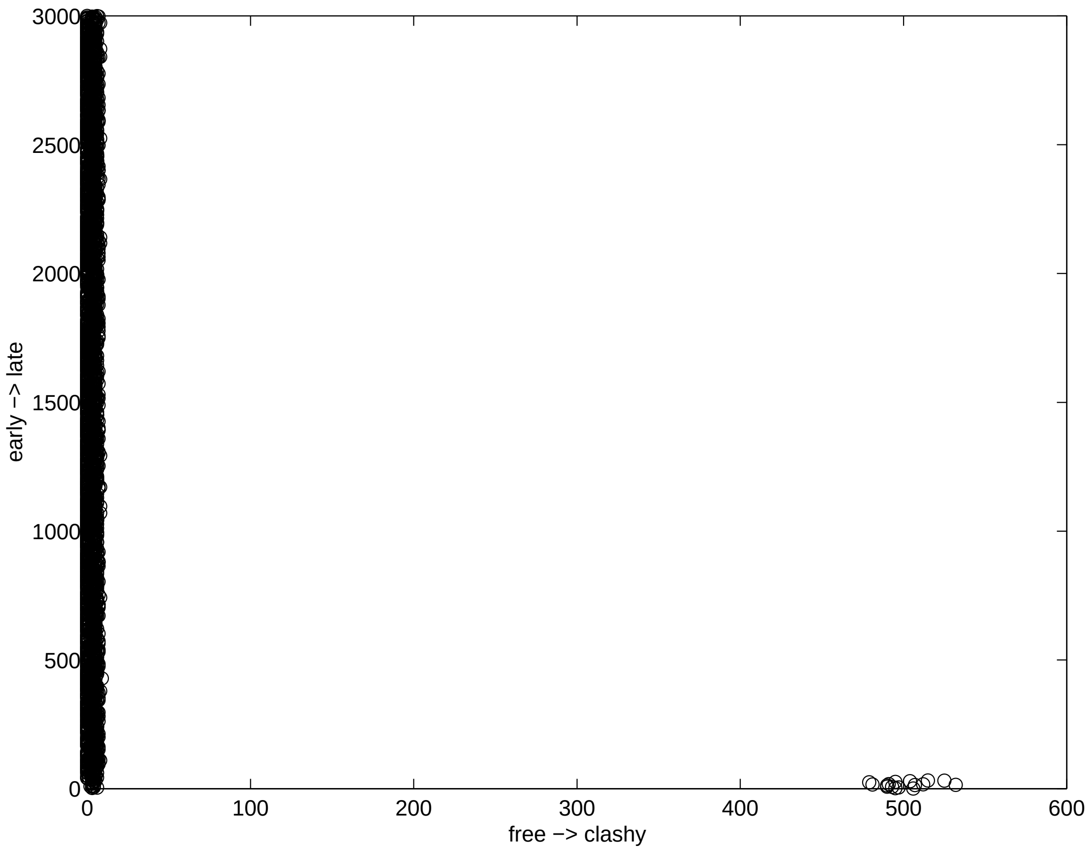
$\text{Prob (green)} = \frac{1}{4}$

Petford and Welsh experimentally obtained on runs of 3-coloring graphs up to 10,000 vertices that about 10 n passes were needed to color the graph.

Anti-Voter Model

Sequentially color the vertices starting from the empty graph.

At each stage color the vertices according to the number of previously colored vertices.


3000
2500
2000
1500
1000
500
0
early −> late
0
100
200
300
400
500
600
free −> clashy

Theorem: (Denley, Reid, Wu) Let G be a 3-colorable graph with n vertices and in which edges appear with probability p exceeding $\sqrt{\frac{1}{2}\frac{ln\,n}{n}}$. Then almost surely, the anti-voter algorithm will produced a proper 3-coloring of G in two passes.

Future Directions:

Random Algorithms for other NP-Hard problems

Generalized Colorings