An Infinite Horizon Manpower Planning Model

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Outline

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Introduction

- The Military Personnel Account (MPA) is the largest account in the Army's budget.
  - In FY2001, the allocation for MPA is $27.7M or 39.6% of the total budget.
- The Army has to carefully manage its inventory of approximately 78K officers.
  - Decide how many officers to access, promote, and, when necessary, separate in order to best meet its desired inventory targets on an annual basis.

Introduction (cont.)

- Office of the Deputy Chief of Staff for Personnel (ODCSPER) is responsible for forecasting and monitoring the officer inventory.
  - Main tools: Two spreadsheet models
    - Budget Allocation Resource of Notional Force (BARON)
    - Competitive Category Army Tracking Systems (CCATS)
  - Other models
    - Officer Projection Aggregate Level or OPAL (1992)
    - Officer Aggregate (OA) System (1998)

Introduction (cont.)

- Concerns with BARON and CCATS:
  - Neither provides management control decisions such as accessions and promotions that best meet the inventory (or strength) targets.
  - Neither incorporates the recently implemented Officer Professional Management System XXI (OPMS XXI) which separates officers with a rank of MAJ or higher into four career fields.
  - Neither accounts for end effects.
    - End effects = errors from having too short a planning horizon.

Introduction (cont.)

- Desirable properties for a model
  - Provide macro level management control decisions that best meet the inventory targets.
  - Incorporate OPMS XXI.
  - Account for end effects.
  - Provide a solution in a reasonable time (e.g., a few minutes).
  - Implementable with a spreadsheet on a PC.
    - Reduce training time and cost.

Background

- In developing our model, we focus on officers in the Army Competitive Category (ACC).
  - Officers in the 16 basic branches, e.g., infantry, armor, and field artillery.
  - Approximately 45K officers (or 58% of the total) are in ACC.
- Under OPMSXXI, officers with ranks of Majors and higher are separated into four career fields:
  - Operations (OP)
  - Operations Support (OS)
  - Information Operation (IO)
  - Institutional Support (IS)

Background (cont.)

- Accession
  - All accessions are at the rank of 2LT.
- Promotion
  - Promotion from 2LT to 1LT is decentralized and nearly automatic.
  - Promotion to CPT and higher ranks
    - Centralized: There is a promotion board for each rank.
    - Three promotion zones
      - Primary zone
      - Above and below the primary zone
    - Comply with the 1980 Defense Officer Personnel Management Act or DOPMA

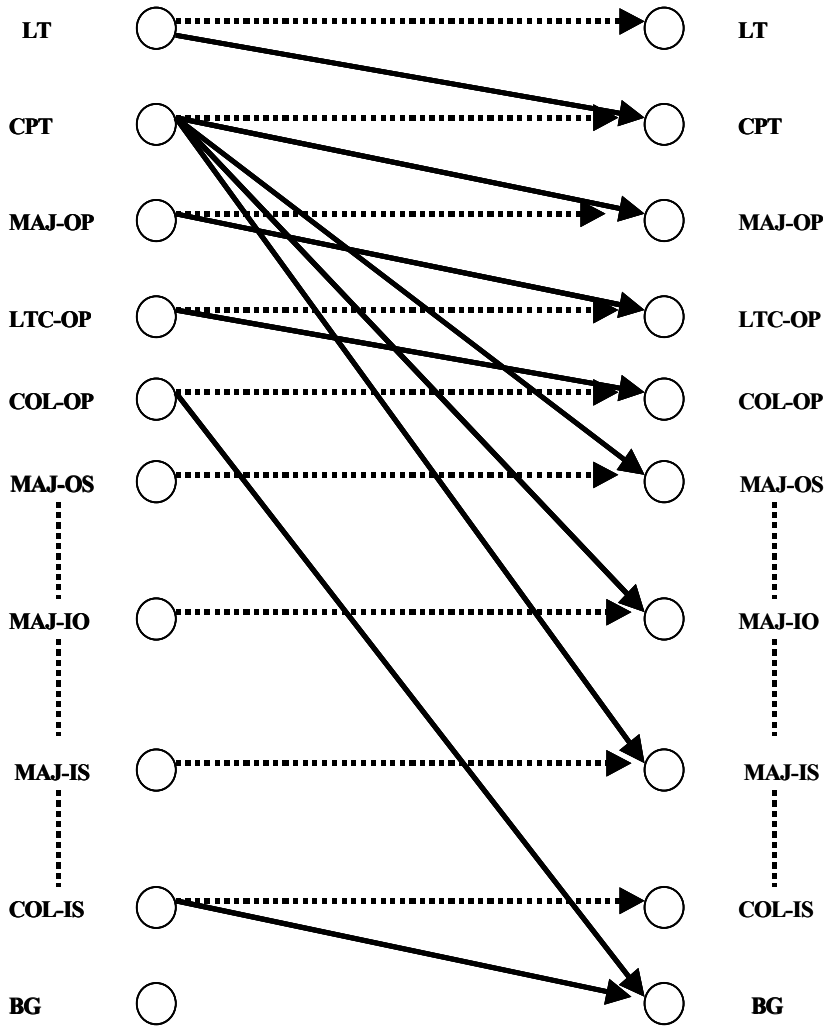
Background (cont.)

- Unmanaged losses
  - Attrition and retirement
- Managed losses (PML) consist of programs that encourage officers to leave the Army before completing 20 years of service.
  - Voluntary Separation Incentive Program
  - Selective Early Retirement Program
  - Voluntary Early Release Program
- Management Goals
  - The Personnel Manning Authorization Document sets target numbers of officers in each grade and competitive category.

Model

- Modeling Goals
  - Small number of variables and constraints.
  - Provide solutions quickly on a PC.
- Assumptions
  - Track inventory at the end of each year
  - Combined 2LT and 1LT into one rank, LT.
  - Do not separate officers by their year groups.
  - The proportion of officers eligible for promotion in each zone is the same every year.

Model (cont.): Personnel Flow

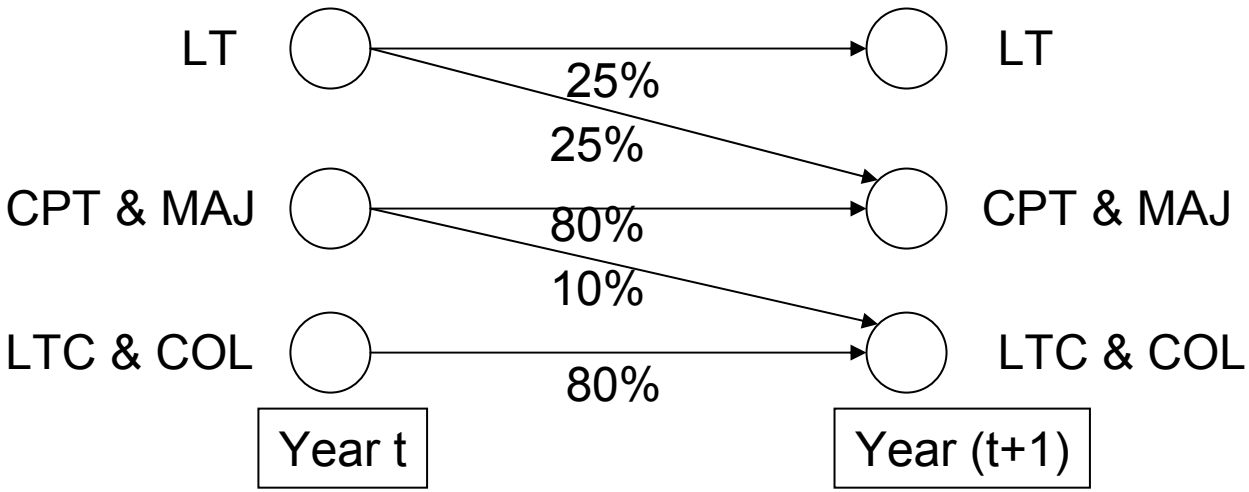


Model (cont)

- Optimization Model:
  - Decision Variables
    - Number of officers in each rank and career field combination to access, promote in each zone, and separate each year.
  - Objective: Minimize (discounted) deviations from targets.
  - Constraints:
    - Balance inventory at end of each year and for each rank and career field combination.
    - DOPMA: Primary, above and below zone promotion.
    - Career field accessions
    - 'Rolldown' requirement
    - Bounds on managed losses
    - Bounds on accessions.

End Effects

- Grinold (1983) refers to errors resulting from having too short a planning horizon as 'end effects'.
- Example:
  - Personnel Flows (attrition and accession not shown)



End Effects (cont.)

- New officers enter the system as LT.
- No PML
- Targets
  - Targets for year 6 and beyond are the same as those in year 5.

	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
LT	75	80	90	100	120
CPT&MAJ	100	110	120	130	150
LTC&COL	90	90	90	95	100

End Effects (cont.)

- Results: Planning horizons of different lengths yield different solutions.
  - True length = infinite

Planning Horizon	Opt. access 1st yr	Opt. access 2nd yr
5 yrs	82	74
10 yrs	91	82
15 yrs	93	83
20 yrs	94	84

Approximation Schemes

- Methods for obtaining approximate solutions to problems with infinite planning horizon.
  - Truncation
  - Primal Equilibrium
  - Dual Equilibrium
  - Sampling

Approximation Schemes (cont.)

- Truncation
  - Most common
  - Assume that the planning horizon ends after T years, where T is, e.g., 5, 10, 15, or 20 years.
  - Ignore end effects.
    - End effects decrease as T increases.
  - Large T results in models with a large number of variables and constraints which, in turn, means a longer solution time.

Approximation Schemes (cont.)

- Primal Equilibrium:
  - Assume that the decision variable reaches equilibrium after *T* years, i.e.,

$$x_t = x_T, \quad \forall t \geq T$$

  - Using the following fact, the infinite horizon problem reduces to a problem with a finite number of variables and constraints

$$\sum_{t=T}^{\infty} \alpha_t = \alpha^T / (1-\alpha), \quad \forall \alpha \in [0,1)$$

Approximation Schemes (cont.)

- Dual Equilibrium
  - Aggregate variables for periods $t \geq T$ into a single variable, i.e.,

$$\hat{x}_T = \sum_{t=T}^{\infty} (1-\alpha)\alpha^{t-T} x_t, \text{ where } \alpha \in [0,1)$$

  - Similarly, aggregate constraints for periods $t \geq T$, into a single constraint

$$\sum_i a_{it} x_i \leq b_t, \forall t \geq T \Rightarrow \sum_i \hat{a}_i x_i \leq \hat{b}$$

Approximation Schemes (cont.)

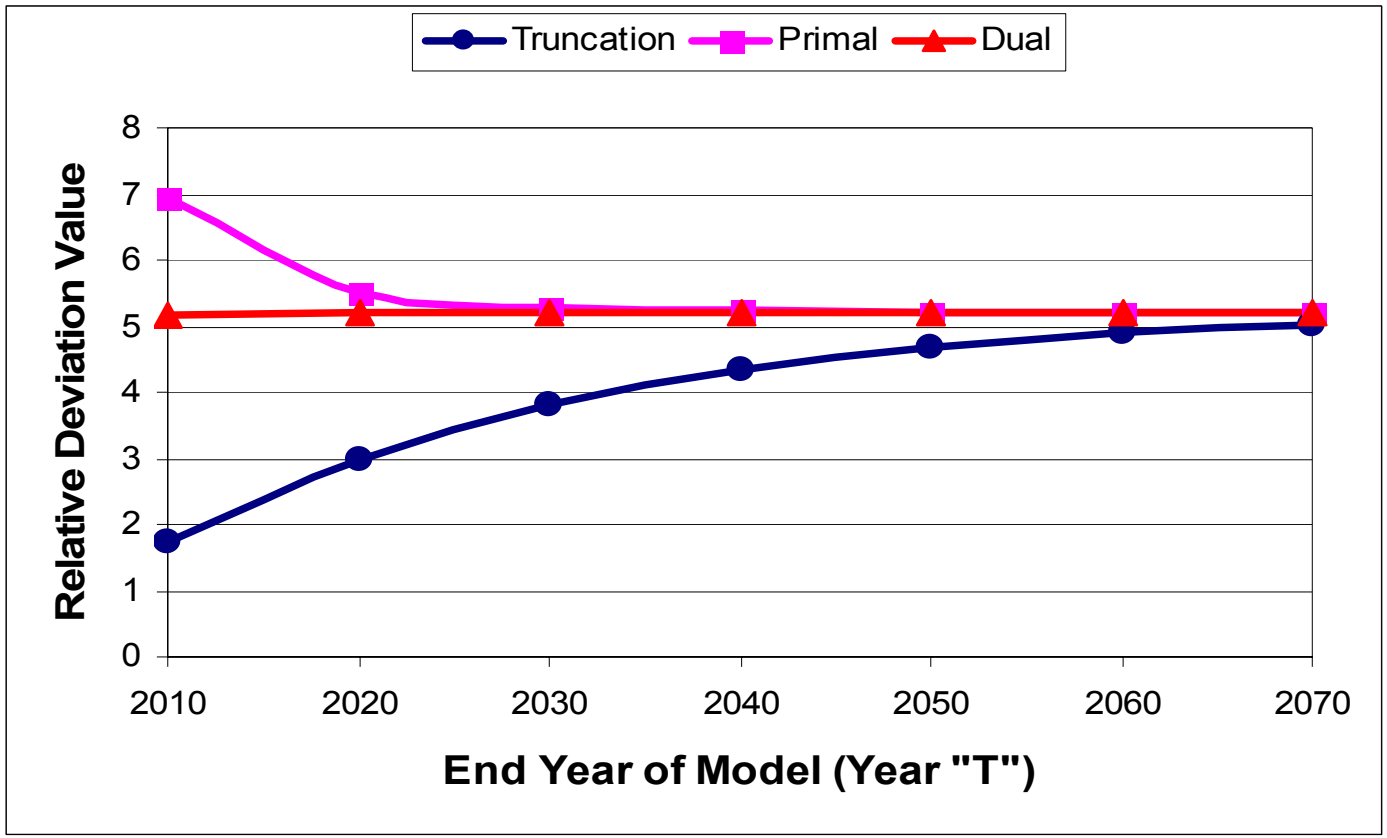
- Sampling: Decision variables for periods not in the sample are assumed to be the same as those in the sample.
  - Normally used in combination with the first three techniques.
  - Example:
    - Consider the following sample of years in a planning horizon: {1, 2, 3, 4, 5, 10, 15, 20}
    - Decision variables for years 6 to 9 are assumed to have the same values as those in year 5.
    - Similar assumptions are true for years 11 to 14, 16 to 19, and 21 to ∞.

Numerical Results

- Input data from ODCSPER
  - Strength targets from FY2001 to FY 2008
  - Initial officer inventories
  - Attrition rates
  - Bounds on promotion and accession rates
- The observations below are based on this set of data.

Numerical Results (cont.)

- Typical convergence behavior



Numerical Results (cont.)

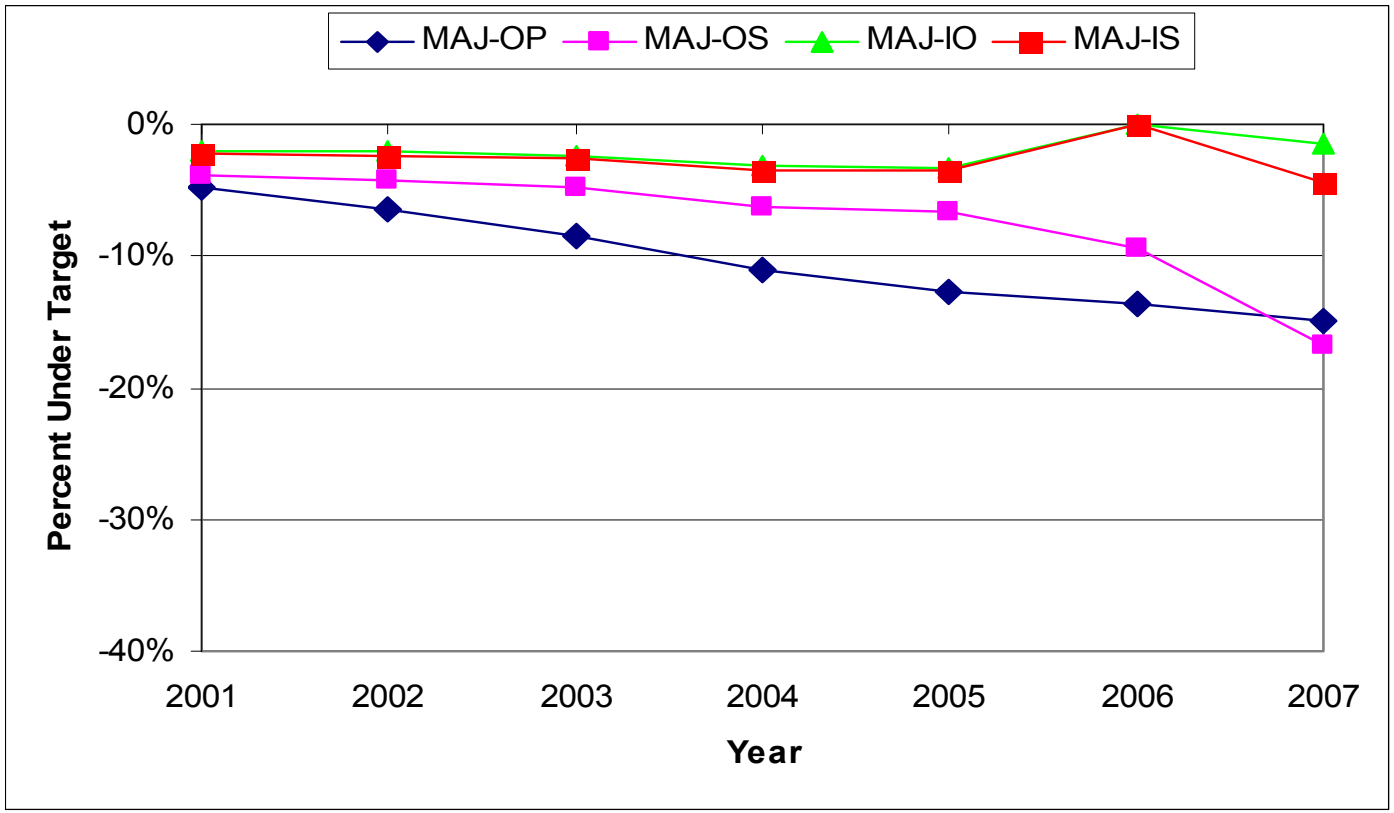
- Dual Equilibrium is the most robust.
  - Provide the best approximate solution
    - Max error = 0.26%
  - Does not require large *T* value.
    - The resulting problem is relatively small.

Numerical Results (cont.)

- Primal Equilibrium
  - Generate solutions with quality similar to those from the truncation technique.
  - When combined with an appropriate sampling scheme (every 5 years) and using a sufficiently large value for T ($\approx$ 30 years), primal equilibrium is comparable to dual equilibrium.

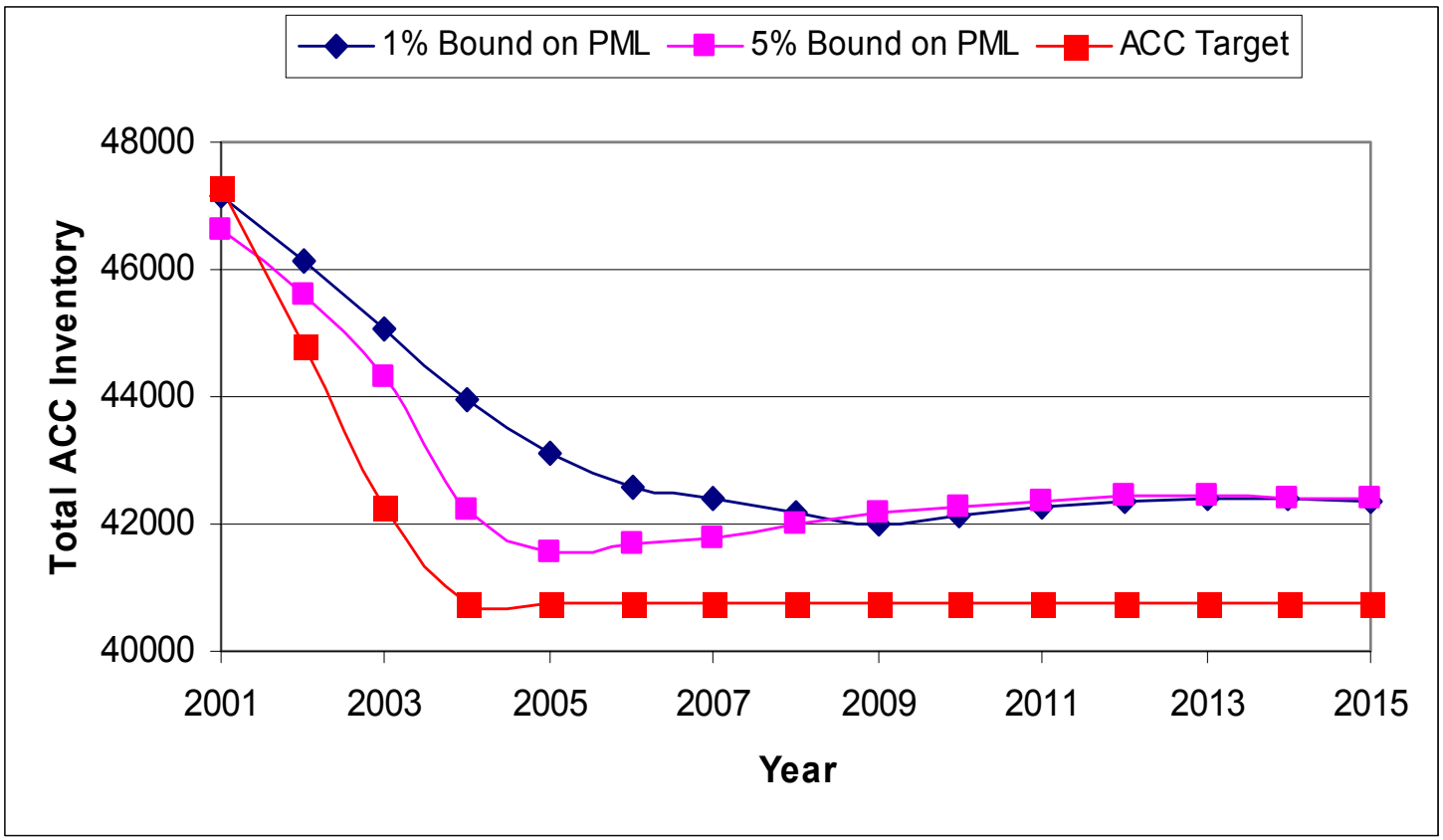
Applications

- Consequence of a new force structure

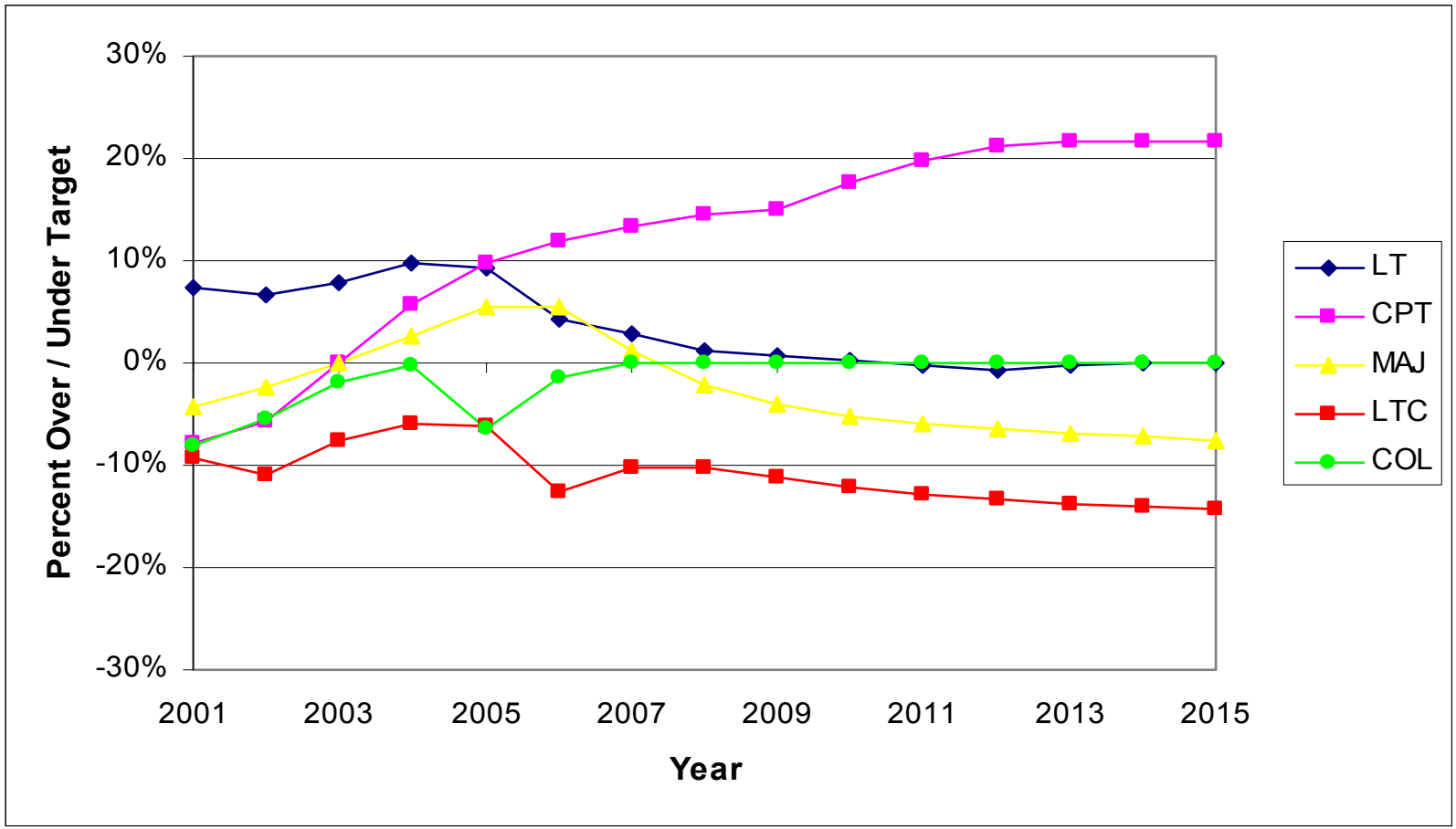


Applications (cont)

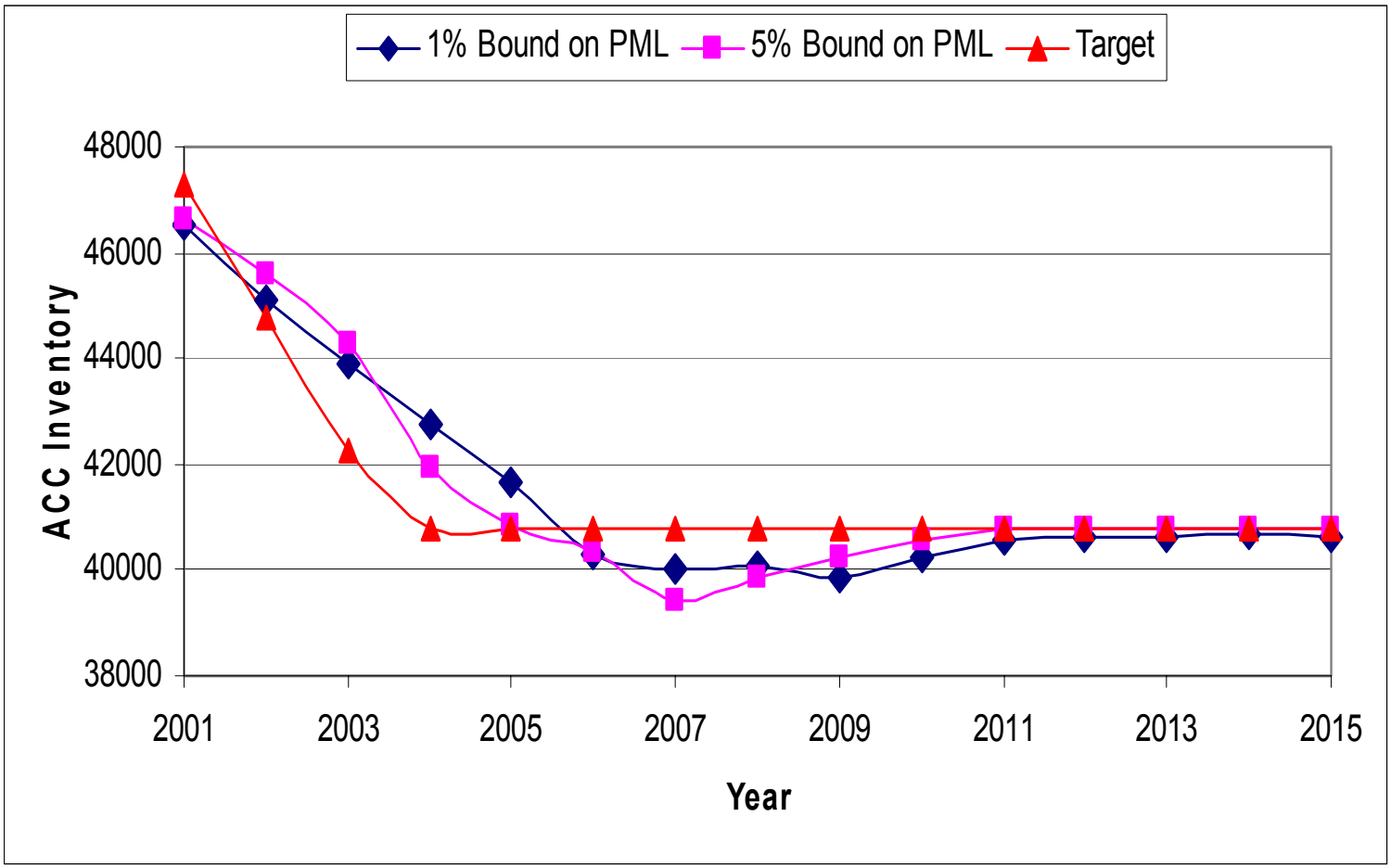
- Effects of reducing the force structure.



Applications (cont)



Applications (cont)



Conclusions and Future Research

- Although the truncation method is used method in many manpower models, it do not account for end effects and, if truncated too early, can lead to wrong decisions, especially during the early part of the planning horizon.
- Use dual equilibrium by itself or primal equilibrium with sampling to approximate the infinite horizon problem.
- Areas of future research
  - Relax the assumption on the attrition rates
    - Stochastic instead of deterministic.
  - Interaction between attrition, promotion, and accession
  - How to model promotion more accurately
    - Year groups
    - Promotion rates (small fluctuations)